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**Final Report of Medical Image Processing**

1. Solution:

In problem 1, the observation is generated by SPECT, of which measurement data is generated from Poisson distribution, . In this case, the probability function is .

The radio activity of each pixels , the probability of obey the Poisson distribution:

The likelihood of **x** is

And log-likelihood is

In E-step, the expectation of log-likelihood is:

In M-step, search to maximize:

In this case,

The pseudo code in MATLAB should be:

Where k is the number of current iteration, diag() returns a square diagonal matix with the elements of vector v on the main diagonal, is the transpose of a matrix or vector, is right divide, which divides each element of left matrix or vector by the corresponding elements of right matrix or vector, and sum() returns the sum of elements of the matrix along the first array dimension whose size does not equal 1.

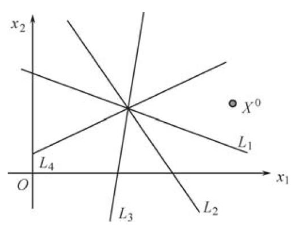
Set the initial image :

Using pinv() function to calculate the pseudo inverse of and the result of can be obtained:

Which is quite close to the result of the third iteration obtained using EM-ML algorithm.

1. Solution:

In this problem, the ART algorithm is used to solve the system of linear equation.



ART algorithm considers one ray sum at a time and can be expressed as:

Where performs the forward projection along the th projection ray, is the measured projection from the th projection bin, is the sum of the squared weights along the th ray.

**(Apologies for it is difficult to denote the right angle of each red line, but all the red line is intended to put vertically to the crossing line)**

With the first order (a) :

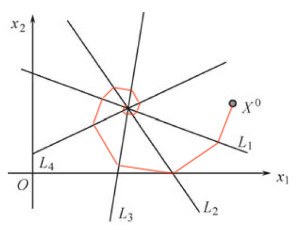


Figure.1.

And the second order (b) :

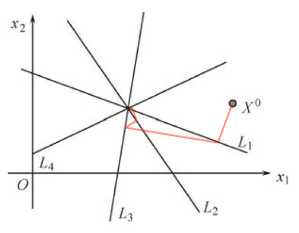


Figure.2.

As the two figures show, the second order performs better in the terms of convergence rate since the expected solution is found with less iterations (each red line represents one iteration) following the second order.

1. Explain three representative reconstruction algorithms (FBP, SVD, ML-EM) in the reconstruction study and compare Pros and Cons of each algorithm.
2. **Filtered Backprojection (FBP) algorithm**:

Backprojection is a process that sums the data from all the projections views. In an image reconstruction problem using tomography, an estimation of specific object from a number of projections is expected. Firstly, the projection of an object is a set of line integrals, resulting from the tomographic measurement process at a given view . The method used to reconstruct the image by summing average attenuated value the along the projection direction.

The mathematical expression of FBP can be:

The objective function can be expressed in the formula for the 2-dimensional inverse Fourier transform:

In polar coordinates,

Using the property:

The integral can be expressed as:

Where .

A value of with a given value of is corresponding to every point in the image plane.

1. **Singular Value Decomposition (SVD):**

Image reconstruction can be obtained by solving a system of linear equations. The image pixels and all projections are labeled in a 1-dimensional sequential order. The system can be rewritten in the matrix form as:

Where , and is the coefficient matrix of the system. The element in represents the weight of the contribution of th pixel to the th projection . However, the reconstruction cannot be done by obtaining the inverse of A because the matrix A is not square normally. A **least-squares** solution can be found in two cases.

In the overdetermined case, where the dimension of measurement data space is more than the dimension of object space, the reconstruction can be calculated by:

In the under-determined case, where the dimension of measurement data space is less than the dimension of object space, the reconstruction can be calculated by:

However, when A is not full rank, cannot be calculated. But a pseudo-inverse of A can be found using **Singular Value Decomposition**.

The mathematical expression of SVD is shown in the following:

Set matrix is denoted as The matrix can be decomposed into:

Where , and

With the singular values arranged in the descending order:

The pseudo-inverse is defined as:

Where

Where the diagonal matrix is defined as:

Thus, the reconstructed image is given as:

1. **ML-EM algorithms:**

The image reconstruction can be done by using the Poisson probability model and minimizing an objective function. The objective function of ML-EM algorithm can be a likelihood function, which is the joint probability density function of Poisson random variables.

In “E” step, the expectation value of the likelihood function is taken for simplifying than taking partial derivatives; in “M” step, the maximum of the expected likelihood function should be found.

The mathematical expression of ML-EM algorithm is shown in the following:

The radio activity of each pixels , the probability of obey the Poisson distribution:

The likelihood of **x** is

And log-likelihood is

In E-step, the expectation of log-likelihood is:

In M-step, search to maximize:

The Pros and Cons of FBP, SVD, and ML-EM algorithms are shown in the Table.1.

Table.1 Comparison of FBP, SVD, ML-EM

|  |  |  |  |
| --- | --- | --- | --- |
|  | FBP  (Filtered Backprojection) | SVD | ML-EM |
| Pros | FBP is computationally efficient.  Spatial resolution can be enhanced using different filters. | SVD allows more complex reconstruction of image because it is easier to solve.  The computation of SVD can be done in limited memory capacity.  SVD provides a more accurate image reconstruction. | The objective function of ML-EM can incorporate the noise in the measurement; thus the noise can be controlled.  It provides a more accurate reconstruction.  ML-EM algorithm is flexible in improving reconstructed image quality. |
| Cons | The filtering step amplifies the noise.  FBP is hard to provide high-quality reconstructed images.  FBP assumes that the object can be imaged perfectly using sinogram representation, which is impossible in practice. | SVD is computationally inefficient.  SVD is computationally expensive. | ML-EM is computationally inefficient.  ML-EM typically converges to a local optimum, and often gets stuck in local optima. |